'LONGITUDINAL NATURE' OF ANTISYMMETRIC TENSOR FIELD AFTER QUANTIZATION AND IMPORTANCE OF THE NORMALIZATION

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It has long been claimed that the antisymmetric tensor field of the second rank is pure longitudinal after quantization. In my opinion, such a situation is quite unacceptable. I repeat the well-known procedure of the derivation of the set of Proca equations. It is shown that it can be written in various forms. Furthermore, on the basis of the Lagrangian formalism I calculate dynamical invariants (including the Pauli-Lubanski vector of relativistic spin for this field). Even at the classical level the Pauli-Lubanski vector can be equal to zero after applications of well-known constraints. The importance of the normalization is pointed out for the problem of the description of quantized fields of maximal spin 1. The correct quantization procedure permits us to propose a solution of this puzzle in the modern field theory. Finally, the discussion of the connection of the Ogievetskiĭ-Polubarinov-Kalb-Ramond field and electrodynamic gauge is presented.
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1. Introduction

Quantum electrodynamics (QED) is a construct which has found overwhelming experimental confirmations (for recent reviews see, *e.g.*, refs. [1,2]). Nevertheless, a number of theoretical aspects of this theory deserve more attention. First of all, they are: the problem of "fictious photons of helicity other than $\pm j$, as well as the indefinite metric that must accompany them"; the renormalization idea, which "would be sensible only if it was applied with finite renormalization factors, not infinite ones (one is not allowed to neglect [and to subtract] infinitely large quantities)"; contradictions with the Weinberg theorem "that no symmetric tensor field of rank j can be constructed from the creation and annihilation operators of massless particles of spin j", *etc.* They were shown by Dirac [3,4] and by Weinberg [5].

Moreover, it appears now that we do not yet understand many specific features of classical electromagnetism; first of all, the problems of longitudinal modes, of the gauge, of the Coulomb action-at-a-distance, and of the Horwitz' additional invariant parameter, refs. [6,7,8,9,10,11,12,13,14,15]. Secondly, the standard model, which has been constructed on the basis of ideas, which are similar to QED, appears to be unable to explain many puzzles in neutrino physics.

In my opinion, all these shortcomings can be the consequence of ignoring several important questions. "In the classical electrodynamics of charged particles, a knowledge of $F^{\mu\nu}$ completely determines the properties of the system. A knowledge of A^μ is redundant there, because it is determined only up to gauge transformations, which do not affect $F^{\mu\nu}$... Such is not the case in quantum theory..." [16]. We learnt, indeed, about this fact from the Aharonov-Bohm [17] and the Aharonov-Casher effects [18]. However, recently several attempts have been undertaken to explain the Aharonov-Bohm effect classically [19]. These attempts have, in my opinion, logical basis. In the meantime, quantizing the antisymmetric tensor field led us to a new puzzle, which until now had not received fair hearings. It was claimed that the antisymmetric tensor field of the second rank is longitudinal after quantization (in the sense of the helicity $\sigma = 0$), refs. [20,21,22,23,24].[*] In the meantime, we know that the antisymmetric tensor field (electric and magnetic fields, indeed) is transverse in the Maxwellian classical electrodynamics. It is not clear how physically longitudinal components can be transformed into the physically transverse ones in some limit. It may be of interest to compare this question with the group-theoretical consideration in ref. [25] which deals with the reduction of rotational degrees of freedom to gauge degrees of freedom in infinite-momentum/zero-mass limit. See the only mentions of the transversality of the quantized antisymmetric tensor field in refs. [26,27]. It is often concluded: one is not allowed to use the antisymmetric tensor field to represent the quantized electromagnetic field in relativistic quantum mechanics. Instead one should pay attention to the 4-vector potential and gauge freedom. Nevertheless, I am convinced that a reliable theory should be constructed on the basis of a minimal number of ingredients ("Occam's Razor") and should have a well-defined classical limit (as well as massless limit). Moreover, physicists recently turned again to the problem of energy in CED [28,29]. Therefore, in this paper I undertake a detailed analysis of translational and rotational properties of the antisymmetric tensor field, I derive *various* forms of the Proca equations (which also can be written in

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[*]M. Kalb and P. Ramond claimed explicitly [21b, p. 2283, the third line from below]: "thus, the massless $\phi_{\mu\nu}$ has one degree of freedom". While they call $\phi_{\mu\nu}$ as "potentials" for the field $F^{\alpha\beta\gamma} = \partial^\alpha\phi^{\beta\gamma} + \partial^\beta\phi^{\gamma\alpha} + \partial^\gamma\phi^{\alpha\beta}$, nevertheless, the physical content of the antisymmetric tensor field of the second rank (the representation $(1,0)\oplus(0,1)$ of the Lorentz group) must be in accordance with the requirements of relativistic invariance. Furthermore, "the helicity – the projection of the spin onto the direction of motion – proves to be equal to zero ... even without the restriction to plane waves, the 3-vector of spin [formula (12) of [23]] vanishes on solutions ...", ref. [23b], Avdeev and Chizhov claimed in their turn.

the Duffin-Kemmer form), then calculate the Pauli-Lubanski operator of relativistic spin (which must define whether the quantum is in the left- or right- polarized states or in the unpolarized state) and then conclude, if it is possible to obtain the conventional electromagnetic theory with photon helicities $\sigma = \pm 1$ provided that strengths (*not* potentials) are chosen to be physical variables in the Lagrangian formalism. The particular case also exists when the Pauli-Lubanski vector for the antisymmetric tensor field of the second rank is equal to zero, that corresponds to the claimed '*longitudinality*' (helicity $\sigma = 0$?) of this field. The answer achieved is that the physical results *depend* on the normalization and chosen type of 'gauge' freedom.

Research in this area from a viewpoint of the Weinberg's $2(2j+1)$ component theory have been started in refs. [30,31,32,8,9,10,11,12,33,34]. I would also like to point out that the problem at hand is directly connected with our understanding of the nature of neutral particles, including neutrinos [35,36,37,38]. From a mathematical viewpoint, theoretical content provided by the space-time structure and corresponding symmetries should not depend on what representation space, which field operators transform on, is chosen.

2. Bargmann-Wigner Procedure, the Proca Equations and Relevant Field Functions

We believe in the power of the group-theoretical methods in the analyses of the physical behaviour of different-type classical (and quantum) fields. We also believe that the Dirac equation can be applied to some particular quantum states of the spin 1/2. Finally, we believe that the spin-0 and spin-1 particles can be constructed by taking the direct product of the spin-1/2 field functions [39]. So, on the basis of these postulates let us firstly repeat the Bargmann-Wigner procedure of obtaining the equations for bosons of spin 0 and 1. The set of basic equations for $j=0$ and $j=1$ are written, e.g., ref. [40]

$$\left[i\gamma^\mu \partial_\mu - m\right]_{\alpha\beta} \Psi_{\beta\gamma}(x) = 0 \quad , \tag{1}$$

$$\left[i\gamma^\mu \partial_\mu - m\right]_{\gamma\beta} \Psi_{\alpha\beta}(x) = 0 \quad . \tag{2}$$

We expand the 4×4 matrix wave function into the antisymmetric and symmetric parts in a standard way

$$\Psi_{[\alpha\beta]} = R_{\alpha\beta}\phi + \gamma^5_{\alpha\delta} R_{\delta\beta} \widetilde{\phi} + \gamma^5_{\alpha\delta} \gamma^\mu_{\delta\tau} R_{\tau\beta} \widetilde{A}_\mu \quad , \tag{3}$$

$$\Psi_{\{\alpha\beta\}} = \gamma^\mu_{\alpha\delta} R_{\delta\beta} A_\mu + \sigma^{\mu\nu}_{\alpha\delta} R_{\delta\beta} F_{\mu\nu} \quad , \tag{4}$$

where $R = CP$ has the properties (which are necessary to make expansions (3,4) to be possible in such a form)

$$R^T = -R \quad , \quad R^\dagger = R = R^{-1} \quad , \tag{5}$$

$$R^{-1}\gamma^5 R = (\gamma^5)^T \quad , \tag{6}$$
$$R^{-1}\gamma^\mu R = -(\gamma^\mu)^T \quad , \tag{7}$$
$$R^{-1}\sigma^{\mu\nu} R = -(\sigma^{\mu\nu})^T \quad . \tag{8}$$

The explicit form of this matrix can be chosen:

$$R = \begin{pmatrix} i\Theta & 0 \\ 0 & -i\Theta \end{pmatrix} \quad , \quad \Theta = -i\sigma_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \tag{9}$$

provided that γ^μ matrices are in the Weyl representation. The equations (1,2) lead to the Kemmer set of the $j = 0$ equations:

$$m\phi = 0 \quad , \tag{10}$$
$$m\widetilde{\phi} = -i\partial_\mu \widetilde{A}^\mu \quad , \tag{11}$$
$$m\widetilde{A}^\mu = -i\partial^\mu \widetilde{\phi} \quad , \tag{12}$$

and to the Proca-Duffin-Kemmer set of the equations for the $j = 1$ case: †, ‡

$$\partial_\alpha F^{\alpha\mu} + \frac{m}{2} A^\mu = 0 \quad , \tag{15}$$
$$2mF^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \quad , \tag{16}$$

In the meantime, in the textbooks, the latter set is usually written as (*e.g.*, ref. [41], p. 135)

$$\partial_\alpha F^{\alpha\mu} + m^2 A^\mu = 0 \quad , \tag{17}$$
$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu \quad , \tag{18}$$

The set (17,18) is obtained from (15,16) after the normalization change $A_\mu \to 2mA_\mu$ or $F_{\mu\nu} \to \frac{1}{2m} F_{\mu\nu}$. Of course, one can investigate other sets of equations with different normalization of the $F_{\mu\nu}$ and A_μ fields. Are all these sets of equations equivalent? As we shall see, to answer this question is not trivial. The paper [34a] argued

† We could use another symmetric matrix $\gamma^5\sigma^{\mu\nu}R$ in the expansion of the symmetric spinor of the second rank. In this case the equations will read

$$i\partial_\alpha \widetilde{F}^{\alpha\mu} + \frac{m}{2} B^\mu = 0 \quad , \tag{13}$$
$$2im\widetilde{F}^{\mu\nu} = \partial^\mu B^\nu - \partial^\nu B^\mu \quad . \tag{14}$$

in which the dual tensor $\widetilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$ presents, because we used that in the Weyl representation $\gamma^5\sigma^{\mu\nu} = \frac{i}{2}\epsilon^{\mu\nu\rho\sigma}\sigma_{\rho\sigma}$; B^μ is the corresponding vector potential. The equation for the antisymmetric tensor field (which can be obtained from this set) does not change its form (cf. [12,42]) but we see some "renormalization" of the field functions. In general, it is permitted to choose various relative phase factors in the expansion of the symmetric wave function (4) and also consider the matrix term of the form $\gamma^5\sigma^{\mu\nu}$. We shall have additional phase factors in equations relating the physical fields and the 4-vector potentials. They can be absorbed by the redefinition of the potentials/fields (the choice of normalization/phase). The above discussion shows that the dual tensor of the second rank can also be expanded in potentials, as opposed to the opinion of the referee (JPA) of my previous paper.

‡ Recently, after completing this work the paper [43] was brought to our attention. It deals with the redundant components in the $j = 3/2$ spin case. If the claims of that paper are correct we would have to change slightly a verbal terminology which we use to describe the above equations.

that the physical normalization is such that in the massless-limit zero-momentum field functions should vanish in the momentum representation (there are no massless particles at rest). Next, we advocate the following approach: the massless limit can and must be taken in the end of all calculations only, *i. e.*, for physical quantities.

Let us proceed further. In order to be able to answer the question about the behaviour of the spin operator $\mathbf{J}^i = \frac{1}{2}\epsilon^{ijk}J^{jk}$ in the massless limit, one should know the behaviour of the fields $F_{\mu\nu}$ and/or A_μ in the massless limit. We want to analyze the first set (15,16). If one advocates the following definitions [44] (p. 209)

$$\epsilon^\mu(\mathbf{0},+1) = -\frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\i\\0\end{pmatrix}, \quad \epsilon^\mu(\mathbf{0},0) = \begin{pmatrix}0\\0\\0\\1\end{pmatrix}, \quad \epsilon^\mu(\mathbf{0},-1) = \frac{1}{\sqrt{2}}\begin{pmatrix}0\\1\\-i\\0\end{pmatrix}, \tag{19}$$

and ($\widehat{p}_i = p_i / \mid \mathbf{p} \mid,\ \gamma = E_p/m$), ref. [44] (p. 68) or ref. [45] (p. 108),

$$\epsilon^\mu(\mathbf{p},\sigma) = L^\mu{}_\nu(\mathbf{p})\epsilon^\nu(\mathbf{0},\sigma)\,, \tag{20}$$

$$L^0{}_0(\mathbf{p}) = \gamma\,, \quad L^i{}_0(\mathbf{p}) = L^0{}_i(\mathbf{p}) = \widehat{p}_i\sqrt{\gamma^2-1}\,, \tag{21}$$

$$L^i{}_k(\mathbf{p}) = \delta_{ik} + (\gamma-1)\widehat{p}_i\widehat{p}_k \tag{22}$$

for the field operator of the 4-vector potential, ref. [45] (p. 109) or ref. [41] (p. 129) [§, ¶]

$$A^\mu(x) = \sum_{\sigma=0,\pm1}\int \frac{d^3\mathbf{p}}{(2\pi)^3}\frac{1}{2E_p}\left[\epsilon^\mu(\mathbf{p},\sigma)a(\mathbf{p},\sigma)e^{-ip\cdot x} + (\epsilon^\mu(\mathbf{p},\sigma))^c b^\dagger(\mathbf{p},\sigma)e^{+ip\cdot x}\right], \tag{23}$$

the normalization of the wave functions in the momentum representation is thus chosen to the unit, $\epsilon^*_\mu(\mathbf{p},\sigma)\epsilon^\mu(\mathbf{p},\sigma) = -1$.[‖] We observe that in the massless limit all the defined polarization vectors of the momentum space do not have good behaviour; the functions describing spin-1 particles tend to infinity. This is not satisfactory. Nevertheless, after renormalizing the potentials, *e. g.*, $\epsilon^\mu \to u^\mu \equiv m\epsilon^\mu$

§Remember that the invariant integral measure over the Minkowski space for physical particles is

$$\int d^4p\,\delta(p^2-m^2) \equiv \int \frac{d^3\mathbf{p}}{2E_p} \quad , \quad E_p = \sqrt{\mathbf{p}^2+m^2} \quad .$$

Therefore, we use the field operator as in (23). The coefficient $(2\pi)^3$ can be considered at this stage as chosen for convenience. In ref. [44] the factor $1/(2E_p)$ was absorbed in creation/annihilation operators and instead of the field operator (23) the operator was used in which the $\epsilon^\mu(\mathbf{p},\sigma)$ functions for a massive spin-1 particle were substituted by $u^\mu(\mathbf{p},\sigma) = (2E_p)^{-1/2}\epsilon^\mu(\mathbf{p},\sigma)$, which may lead to confusion in the definitions of the massless limit $m \to 0$ for classical polarization vectors.

¶In general, it might be useful to consider front-form helicities (and/or "time-like" polarizations) too. But, we leave the presentation of rigorous theory of this type for subsequent publications.

‖The metric used in this paper $g^{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$ is different from that of ref. [44].

we come to the wave functions in the momentum representation:**

$$u^\mu(\mathbf{p},+1) = -\frac{N}{\sqrt{2}m}\begin{pmatrix} p_r \\ m+\frac{p_1 p_r}{E_p+m} \\ im+\frac{p_2 p_r}{E_p+m} \\ \frac{p_3 p_r}{E_p+m}\end{pmatrix}, \quad u^\mu(\mathbf{p},-1) = \frac{N}{\sqrt{2}m}\begin{pmatrix} p_l \\ m+\frac{p_1 p_l}{E_p+m} \\ -im+\frac{p_2 p_l}{E_p+m} \\ \frac{p_3 p_l}{E_p+m}\end{pmatrix}, \tag{24}$$

$$u^\mu(\mathbf{p},0) = \frac{N}{m}\begin{pmatrix} p_3 \\ \frac{p_1 p_3}{E_p+m} \\ \frac{p_2 p_3}{E_p+m} \\ m+\frac{p_3^2}{E_p+m}\end{pmatrix}, \tag{25}$$

($N = m$ and $p_{r,l} = p_1 \pm ip_2$) which do not diverge in the massless limit. Two of the massless functions (with $\sigma = \pm 1$) are equal to zero when the particle, described by this field, is moving along the third axis ($p_1 = p_2 = 0,\ p_3 \neq 0$). The third one ($\sigma = 0$) is

$$u^\mu(p_3,0)\mid_{m\to 0} = \begin{pmatrix} p_3 \\ 0 \\ 0 \\ \frac{p_3^2}{E_p}\end{pmatrix} \equiv \begin{pmatrix} E_p \\ 0 \\ 0 \\ E_p\end{pmatrix}, \tag{26}$$

and at the rest ($E_p = p_3 \to 0$) also vanishes. Thus, such a field operator describes the "longitudinal photons" which is in complete accordance with the Weinberg theorem $B - A = \sigma$ for massless particles (let us remind that we use the $D(1/2,1/2)$ representation). Thus, the change of the normalization can lead to the "change" of physical content described by the classical field (at least, comparing with the well-accepted one). Of course, in the quantum case one should somehow fix the form of commutation relations by some physical principles.* In the connection with the above consideration it is interesting to remind that the authors of ref. [41] (see page 136 therein) tried to inforce the Stueckelberg's Lagrangian in order to overcome the difficulties related to the $m \to 0$ limit (or the Proca theory $\to$ Quantum Electrodynamics). The Stueckelberg's Lagrangian is well known to contain the additional term which may be put in correspondence to some scalar (longitudinal) field (cf. also [6]).

Furthermore, it is easy to prove that the physical fields $F^{\mu\nu}$ (defined as in (15,16), for instance) vanish in the massless zero-momentum limit under the both definitions of normalization and field equations. It is straightforward to find $\mathbf{B}^{(+)}(\mathbf{p},\sigma) = \frac{i}{2m}\mathbf{p}\times\mathbf{u}(\mathbf{p},\sigma)$, $\mathbf{E}^{(+)}(\mathbf{p},\sigma) = \frac{i}{2m}p_0\mathbf{u}(\mathbf{p},\sigma) - \frac{i}{2m}\mathbf{p}u^0(\mathbf{p},\sigma)$ and the corresponding

**It is interesting to note that all the vectors u^μ satisfy the condition $p_\mu u^\mu(\mathbf{p},\sigma) = 0$. It is relevant to the case of the Lorentz gauge and, perhaps, to the analyses of the neutrino theories of light.

*I am *very* grateful to the anonymous referee of my previous papers ("Foundation of Physics") who suggested to fix them by requirements of the dimensionless nature of the action (apart from the requirements of the translational and rotational invariancies).

negative-energy strengths. Here they are:[†]

$$\mathbf{B}^{(+)}(\mathbf{p},+1) = -\frac{iN}{2\sqrt{2}m}\begin{pmatrix} -ip_3 \\ p_3 \\ ip_r \end{pmatrix} = +e^{-i\alpha_{-1}}\mathbf{B}^{(-)}(\mathbf{p},-1) \quad , \tag{27}$$

$$\mathbf{B}^{(+)}(\mathbf{p},0) = \frac{iN}{2m}\begin{pmatrix} p_2 \\ -p_1 \\ 0 \end{pmatrix} = -e^{-i\alpha_0}\mathbf{B}^{(-)}(\mathbf{p},0) \quad , \tag{28}$$

$$\mathbf{B}^{(+)}(\mathbf{p},-1) = \frac{iN}{2\sqrt{2}m}\begin{pmatrix} ip_3 \\ p_3 \\ -ip_l \end{pmatrix} = +e^{-i\alpha_{+1}}\mathbf{B}^{(-)}(\mathbf{p},+1) \quad , \tag{29}$$

and

$$\mathbf{E}^{(+)}(\mathbf{p},+1) = -\frac{iN}{2\sqrt{2}m}\begin{pmatrix} E_p - \frac{p_1 p_r}{E_p+m} \\ iE_p - \frac{p_2 p_r}{E_p+m} \\ -\frac{p_3 p_r}{E+m} \end{pmatrix} = +e^{-i\alpha'_{-1}}\mathbf{E}^{(-)}(\mathbf{p},-1) \quad , \tag{30}$$

$$\mathbf{E}^{(+)}(\mathbf{p},0) = \frac{iN}{2m}\begin{pmatrix} -\frac{p_1 p_3}{E_p+m} \\ -\frac{p_2 p_3}{E_p+m} \\ E_p - \frac{p_3^2}{E_p+m} \end{pmatrix} = -e^{-i\alpha'_0}\mathbf{E}^{(-)}(\mathbf{p},0) \quad , \tag{31}$$

$$\mathbf{E}^{(+)}(\mathbf{p},-1) = \frac{iN}{2\sqrt{2}m}\begin{pmatrix} E_p - \frac{p_1 p_l}{E_p+m} \\ -iE_p - \frac{p_2 p_l}{E_p+m} \\ -\frac{p_3 p_l}{E_p+m} \end{pmatrix} = +e^{-i\alpha'_{+1}}\mathbf{E}^{(-)}(\mathbf{p},+1) \quad , \tag{32}$$

where we denoted, as previously, a normalization factor appearing in the definitions of the potentials (and/or in the definitions of the physical fields through potentials) as N. Let us note that as a result of the above definitions we have

- The cross products of magnetic fields of different spin states (such as $\mathbf{B}^{(+)}(\mathbf{p},\sigma)\times \mathbf{B}^{(-)}(\mathbf{p},\sigma'))$ may be unequal to zero and may be expressed by the "time-like" potential (see the formula (43) below):[‡]

$$\mathbf{B}^{(+)}(\mathbf{p},+1)\times\mathbf{B}^{(-)}(\mathbf{p},+1) = -\frac{iN^2}{4m^2}p_3\begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} = $$
$$= -\mathbf{B}^{(+)}(\mathbf{p},-1)\times\mathbf{B}^{(-)}(\mathbf{p},-1)\,, \tag{33}$$

$$\mathbf{B}^{(+)}(\mathbf{p},+1)\times\mathbf{B}^{(-)}(\mathbf{p},0) = -\frac{iN^2}{4m^2}\frac{p_r}{\sqrt{2}}\begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} = $$
$$= +\mathbf{B}^{(+)}(\mathbf{p},0)\times\mathbf{B}^{(-)}(\mathbf{p},-1)\,, \tag{34}$$

[†]In this paper we assume that $[\epsilon^\mu(\mathbf{p},\sigma)]^c = e^{i\alpha_\sigma}[\epsilon^\mu(\mathbf{p},\sigma)]^*$, with α_σ being arbitrary phase factors at this stage. Thus, $\mathcal{C} = I_{4\times 4}$ and $S^c = \mathcal{K}$. It is interesting to investigate other choices of the $\mathcal{C}$, the charge conjugation matrix and/or consider a field operator composed of CP-conjugate states.

[‡]The relevant phase factors are assumed to be equal to zero here.

$$\mathbf{B}^{(+)}(\mathbf{p},-1)\times\mathbf{B}^{(-)}(\mathbf{p},0) = -\frac{iN^2}{4m^2}\frac{p_l}{\sqrt{2}}\begin{pmatrix}p_1\\p_2\\p_3\end{pmatrix} = = +\mathbf{B}^{(+)}(\mathbf{p},0)\times\mathbf{B}^{(-)}(\mathbf{p},+1)\,. \quad (35)$$

Other cross products are equal to zero.

- Furthermore, one can find the interesting relation:

$$\mathbf{B}^{(+)}(\mathbf{p},+1)\cdot\mathbf{B}^{(-)}(\mathbf{p},+1)+\mathbf{B}^{(+)}(\mathbf{p},-1)\cdot\mathbf{B}^{(-)}(\mathbf{p},-1)+ +\mathbf{B}^{(+)}(\mathbf{p},0)\cdot\mathbf{B}^{(-)}(\mathbf{p},0) = \frac{N^2}{2m^2}(E_p^2-m^2)\,, \quad (36)$$

due to

$$\mathbf{B}^{(+)}(\mathbf{p},+1)\cdot\mathbf{B}^{(-)}(\mathbf{p},+1) = \frac{N^2}{8m^2}(p_r p_l+2p_3^2) = = +\mathbf{B}^{(+)}(\mathbf{p},-1)\cdot\mathbf{B}^{(-)}(\mathbf{p},-1)\,, \quad (37)$$

$$\mathbf{B}^{(+)}(\mathbf{p},+1)\cdot\mathbf{B}^{(-)}(\mathbf{p},0) = \frac{N^2}{4\sqrt{2}m^2}p_3p_r = -\mathbf{B}^{(+)}(\mathbf{p},0)\cdot\mathbf{B}^{(-)}(\mathbf{p},-1)\,, \quad (38)$$

$$\mathbf{B}^{(+)}(\mathbf{p},-1)\cdot\mathbf{B}^{(-)}(\mathbf{p},0) = -\frac{N^2}{4\sqrt{2}m^2}p_3p_l = -\mathbf{B}^{(+)}(\mathbf{p},0)\cdot\mathbf{B}^{(-)}(\mathbf{p},+1)\,, \quad (39)$$

$$\mathbf{B}^{(+)}(\mathbf{p},+1)\cdot\mathbf{B}^{(-)}(\mathbf{p},-1) = \frac{N^2}{8m^2}p_r^2\,, \quad (40)$$

$$\mathbf{B}^{(+)}(\mathbf{p},-1)\cdot\mathbf{B}^{(-)}(\mathbf{p},+1) = \frac{N^2}{8m^2}p_l^2\,, \quad (41)$$

$$\mathbf{B}^{(+)}(\mathbf{p},0)\cdot\mathbf{B}^{(-)}(\mathbf{p},0) = \frac{N^2}{4m^2}p_rp_l\,. \quad (42)$$

For the sake of completeness let us present the fields corresponding to the "time-like" polarization:

$$u^\mu(\mathbf{p},0_t) = \frac{N}{m}\begin{pmatrix}E_p\\p_1\\p_2\\p_3\end{pmatrix}\quad,\quad \mathbf{B}^{(\pm)}(\mathbf{p},0_t)=\mathbf{0}\quad,\quad \mathbf{E}^{(\pm)}(\mathbf{p},0_t)=\mathbf{0}\,. \quad (43)$$

The polarization vector $u^\mu(\mathbf{p},0_t)$ has the good behaviour in $m\to 0$, $N=m$ (and also in the subsequent limit $\mathbf{p}\to\mathbf{0}$) and it may correspond to some quantized field (particle). As one can see, the field operator composed of the states of longitudinal (e.g., as positive-energy solution) and time-like (e.g., as negative-energy solution)[§] polarizations may describe a situation when a particle and an antiparticle have

[§]At the present level of our knowledge only *relative* intrinsic parity has physical sense. Cf. [12].

opposite intrinsic parities (cf. [34a]). Furthermore, in the case of the normalization of potentials to the mass $N = m$ the physical fields $\mathbf{B}$ and $\mathbf{E}$, which correspond to the "time-like" polarization, are equal to zero identically. The longitudinal fields (strengths) are equal to zero in this limit only when one chooses the frame with $p_3 =\mid \mathbf{p} \mid$, cf. with the light front formulation, ref. [46]. In the case $N = 1$ and (15,16) we have, in general, the divergent behaviour of potentials and strengths.¶

3. Translational and Rotational Properties of Antisymmetric Tensor Field.

I begin this Section with the antisymmetric tensor field operator (in general, complex-valued):

$$F^{\mu\nu}(x) \;=\; \sum_{\sigma=0,\pm 1} \int \frac{d^3\mathbf{p}}{(2\pi)^3 2E_p} \left[F^{\mu\nu}_{(+)}(\mathbf{p},\sigma)\, a(\mathbf{p},\sigma)\, e^{-ipx} + F^{\mu\nu}_{(-)}(\mathbf{p},\sigma)\, b^\dagger(\mathbf{p},\sigma)\, e^{+ipx} \right] \tag{44}$$

and with the Lagrangian, including, in general, mass term:‖

$$\mathcal{L} = \frac{1}{4}(\partial_\mu F_{\nu\alpha})(\partial^\mu F^{\nu\alpha}) - \frac{1}{2}(\partial_\mu F^{\mu\alpha})(\partial^\nu F_{\nu\alpha}) - \frac{1}{2}(\partial_\mu F_{\nu\alpha})(\partial^\nu F^{\mu\alpha}) + \frac{1}{4} m^2 F_{\mu\nu} F^{\mu\nu} \,. \tag{46}$$

The Lagrangian leads to the equation of motion in the following form (provided that the appropriate antisymmetrization procedure has been taken into account):

$$\frac{1}{2}(\square + m^2) F_{\mu\nu} + (\partial_\mu F_{\alpha\nu}{}^{,\alpha} - \partial_\nu F_{\alpha\mu}{}^{,\alpha}) = 0 \quad , \tag{47}$$

where $\square = -\partial_\alpha \partial^\alpha$, cf. with the set of equations (15,16). It is this equation for antisymmetric-tensor-field components that follows from the Proca-Duffin-Kemmer-Bargmann-Wigner consideration, provided that $m \neq 0$ and in the final expression one takes into account the Klein-Gordon equation $(\square - m^2)F_{\mu\nu} = 0$. The latter expresses relativistic dispersion relations $E^2 - \mathbf{p}^2 = m^2$ and it follows from the coordinate Lorentz transformation laws [47], §2.3.

¶In the case of $N = 1$ the fields $\mathbf{B}^\pm(\mathbf{p}, 0_t)$ and $\mathbf{E}^\pm(\mathbf{p}, 0_t)$ would be undefined. This fact was also not fully appreciated in the previous formulations of the theory of $(1,0) \oplus (0,1)$ and $(1/2,1/2)$ fields.

‖The massless limit ($m \to 0$) of the Lagrangian is connected with the Lagrangians used in the conformal field theory and in the conformal supergravity by adding the total derivative:

$$\mathcal{L}_{CFT} = \mathcal{L} + \frac{1}{2}\partial_\mu \left(F_{\nu\alpha}\partial^\nu F^{\mu\alpha} - F^{\mu\alpha}\partial^\nu F_{\nu\alpha} \right) \quad . \tag{45}$$

The Kalb-Ramond gauge-invariant form (with respect to "gauge" transformations $F_{\mu\nu} \to F_{\mu\nu} + \partial_\nu \Lambda_\mu - \partial_\mu \Lambda_\nu$), ref. [20,21], is obtained only if one uses the Fermi procedure *mutatis mutandis* by removing the additional "phase" field $\lambda(\partial_\mu F^{\mu\nu})^2$, with the appropriate coefficient λ, from the Lagrangian. This has certain analogy with the QED, where the question of whether the Lagrangian is gauge-invariant or not, is solved depending on the presence of the term $\lambda(\partial_\mu A^\mu)^2$. For details see ref. [21] and what is below.
In general it is possible to introduce various forms of the mass term and of corresponding normalization of the field.

Following the variation procedure given, *e.g.*, in refs. [48,49,50] one can obtain that the energy-momentum tensor is expressed:

$$\begin{aligned}\Theta^{\lambda\beta} &= \frac{1}{2}\left[(\partial^\lambda F_{\mu\alpha})(\partial^\beta F^{\mu\alpha}) - 2(\partial_\mu F^{\mu\alpha})(\partial^\beta F^{\lambda}{}_{\alpha}) -\right.\\ &- \left. 2(\partial^\mu F^{\lambda\alpha})(\partial^\beta F_{\mu\alpha})\right] - \mathcal{L} g^{\lambda\beta}\,. \end{aligned} \quad (48)$$

One can also obtain that for rotations $x^{\mu'} = x^\mu + \omega^{\mu\nu} x_\nu$ the corresponding variation of the wave function is found from the formula:

$$\delta F^{\alpha\beta} = \frac{1}{2}\omega^{\kappa\tau}\mathcal{T}^{\alpha\beta,\mu\nu}_{\kappa\tau} F_{\mu\nu} \quad . \quad (49)$$

The generators of infinitesimal transformations are then defined as

$$\begin{aligned}\mathcal{T}^{\alpha\beta,\mu\nu}_{\kappa\tau} &= \frac{1}{2} g^{\alpha\mu}(\delta^\beta_\kappa\,\delta^\nu_\tau - \delta^\beta_\tau\,\delta^\nu_\kappa) + \frac{1}{2} g^{\beta\mu}(\delta^\nu_\kappa\delta^\alpha_\tau - \delta^\nu_\tau\,\delta^\alpha_\kappa) +\\ &+ \frac{1}{2} g^{\alpha\nu}(\delta^\mu_\kappa\,\delta^\beta_\tau - \delta^\mu_\tau\,\delta^\beta_\kappa) + \frac{1}{2} g^{\beta\nu}(\delta^\alpha_\kappa\,\delta^\mu_\tau - \delta^\alpha_\tau\,\delta^\mu_\kappa) \quad . \end{aligned} \quad (50)$$

It is $\mathcal{T}^{\alpha\beta,\mu\nu}_{\kappa\tau}$, the generators of infinitesimal transformations, that enter in the formula for the relativistic spin tensor:

$$J_{\kappa\tau} = \int d^3\mathbf{x}\left[\frac{\partial\mathcal{L}}{\partial(\partial F^{\alpha\beta}/\partial t)}\mathcal{T}^{\alpha\beta,\mu\nu}_{\kappa\tau} F_{\mu\nu}\right] \quad . \quad (51)$$

As a result one obtains:

$$\begin{aligned} J_{\kappa\tau} &= \int d^3\mathbf{x}\,[(\partial_\mu F^{\mu\nu})(g_{0\kappa}F_{\nu\tau} - g_{0\tau}F_{\nu\kappa}) - (\partial_\mu F^{\mu}{}_{\kappa})F_{0\tau} + (\partial_\mu F^{\mu}{}_{\tau})F_{0\kappa} +\\ &+ F^{\mu}{}_{\kappa}(\partial_0 F_{\tau\mu} + \partial_\mu F_{0\tau} + \partial_\tau F_{\mu 0}) - F^{\mu}{}_{\tau}(\partial_0 F_{\kappa\mu} + \partial_\mu F_{0\kappa} + \partial_\kappa F_{\mu 0})]\,. \end{aligned} \quad (52)$$

If one agrees that the orbital part of the angular momentum

$$L_{\kappa\tau} = x_\kappa \Theta_{0\,\tau} - x_\tau \Theta_{0\,\kappa} \quad , \quad (53)$$

with $\Theta_{\tau\lambda}$ being the energy-momentum tensor, does not contribute to the Pauli-Lubanski operator when acting on the one-particle free states (as in the Dirac $j = 1/2$ case), then the Pauli-Lubanski 4-vector is constructed as follows [41] (Eq. (2-21))

$$W_\mu = -\frac{1}{2}\epsilon_{\mu\kappa\tau\nu} J^{\kappa\tau} P^\nu \quad , \quad (54)$$

with $J^{\kappa\tau}$ defined by Eqs. (51,52). The 4-momentum operator P^ν can be replaced by its eigenvalue when acting on the plane-wave eigenstates. Furthermore, one should choose space-like normalized vector $n^\mu n_\mu = -1$, for example $n_0 = 0$, $\mathbf{n} = \widehat{\mathbf{p}} = \mathbf{p}/|\mathbf{p}|$. ** After lengthy calculations in a spirit of [41], pp. 58, 147, one can find the

** One should remember that the helicity operator is usually connected with the Pauli-Lubanski vector in the following manner $(\mathbf{J}\cdot\widehat{\mathbf{p}}) = (\mathbf{W}\cdot\widehat{\mathbf{p}})/E_p$, see ref. [51]. The choice of ref. [41], p. 147, $n^\mu = \left(t^\mu - p^\mu \frac{p\cdot t}{m^2}\right)\frac{m}{|\mathbf{p}|}$, with $t^\mu \equiv (1,0,0,0)$ being a time-like vector, is also possible but it leads to some obscurities in the procedure of taking the massless limit. These obscurities will be clarified in a separate paper.

explicit form of the relativistic spin:

$$(W_\mu \cdot n^\mu) = -(\mathbf{W}\cdot\mathbf{n}) = -\frac{1}{2}\epsilon^{ijk} n^k J^{ij} p^0 \quad , \tag{55}$$

$$\mathbf{J}^k = \frac{1}{2}\epsilon^{ijk} J^{ij} = \epsilon^{ijk}\int d^3\mathbf{x}\left[F^{0i}(\partial_\mu F^{\mu j}) + F_\mu{}^j(\partial^0 F^{\mu i} + \partial^\mu F^{i0} + \partial^i F^{0\mu})\right] . \tag{56}$$

Now it becomes obvious that the application of the generalized Lorentz conditions (which are quantum versions of free-space dual Maxwell's equations) leads in such a formulation to the absence of electromagnetism in a conventional sense. The resulting Kalb-Ramond field is longitudinal (helicity $\sigma = 0$). All the components of the angular momentum tensor for this case are identically equated to zero. The discussion of this fact can also be found in ref. [21,9]. This situation can occur in the particular choice of the normalization of the field operators and unusual "gauge" invariance.

Furthermore, the spin operator recasts in the terms of the vector potentials as follows (if one takes into account the dynamical equations, Eqs. (13,14,15,16))††

$$\begin{aligned}\mathbf{J}^k \quad &= \quad \epsilon^{ijk}\int d^3\mathbf{x}\left[F^{0i}(\partial_\mu F^{\mu j}) + \widetilde{F}^{0i}(\partial_\mu \widetilde{F}^{\mu j})\right] = \\ &= \frac{1}{4}\epsilon^{ijk}\int d^3x\left[B^j(\partial^0 B^i - \partial^i B^0) - A^j(\partial^0 A^i - \partial^i A^0)\right] \quad . \end{aligned}\tag{59}$$

If we put, as usual, $\widetilde{F}^{\mu\nu} = \pm i F^{\mu\nu}$ (or $B^\mu = \pm A^\mu$) for the right- and left- circularly polarized radiation we shall again obtain equating the spin operator to zero. The same situation would occur if one chooses "unappropriate" normalization and/or if one uses the equations (17,18) in the massless limit without necessary precautions. The straightforward application of (17,18) would lead to the proportionality $J_{\kappa\tau} \sim m^2$ and, thus, it appears that the spin operator would be equal to zero in the massless limit, provided that the components of A_μ have good behaviour (do not diverge in $m \to 0$). Probably, this fact (the relation between generators and the normalization) was the origin of why many respectable persons claimed that the antisymmetric tensor field is a pure longitudinal field. On the other hand, in a

†† The formula (59) has certain similarities with the formula for the spin vector obtained from Eqs. (5.15,5.21) of ref. [50]:

$$\mathbf{J}_i \quad = \quad \epsilon_{ijk}\int J^0_{jk} d^3\mathbf{x} \quad , \tag{57}$$

$$J^0_{\alpha\beta} \quad = \quad \left(A_\beta \frac{\partial A_\alpha}{\partial x_0} - A_\alpha \frac{\partial A_\beta}{\partial x_0}\right) . \tag{58}$$

It describes the "transverse photons" in the ordinary wisdom. But, not all the questions related with the second B_μ potential, the dual tensor $\widetilde{F}^{\mu\nu}$ and the normalization of 4-potentials and fields have been clarified in the standard textbooks.

private communication Prof. Y. S. Kim stressed that neither he nor E. Wigner used the normalization of the spin generators to the mass. What is the situation which is realized in Nature (or both)? The theoretical answer depends on the choice of the field operators, namely on the choice of positive- and negative- energy solutions, creation/annihilation operators and the normalization.

One of the possible ways to obtain helicities $\sigma = \pm 1$ is a modification of the electromagnetic field tensor, *i.e.*, introducing the non-Abelian electrodynamics [7,52]:

$$F_{\mu\nu} \quad \Rightarrow \quad \mathbf{G}^a_{\mu\nu} = \partial_\mu A^{(a)}_\nu - \partial_\nu A^{(a)}_\mu - i\frac{e}{\hbar}[A^{(b)}_\mu , A^{(c)}_\nu] \quad , \tag{60}$$

where (a), (b), (c) denote the vector components in the (1), (2), (3) circular basis. In other words, one can add some ghost field (the $\mathbf{B}^{(3)}$ field) to the antisymmetric tensor $F_{\mu\nu}$ which initially supposed to contain transverse components only. As a matter of fact this induces hypotheses on a massive photon and/or an additional displacement current. I can agree with the *possibility* of the $\mathbf{B}^{(3)}$ field concept (while it is required *rigorous* elaboration in the terminology of the modern quantum field theory), but, at the moment, I prefer to avoid any auxiliary constructions (even if they may be valuable in intuitive explanations and generalizations). If these non-Abelian constructions exist they should be deduced from a more general theory on the basis of some fundamental postulates, *e.g.*, in a spirit of refs. [53,34,38]. Moreover, this concept appears to be in contradiction with the concept of the $m \to 0$ group contraction for a photon as presented by Wigner and Inonu [54] and Kim [25].

In the procedure of the quantization one can reveal an important case, when the transversality (in the meaning of existence of $\sigma = \pm 1$) of the antisymmetric tensor field is preserved. This conclusion is related with existence of the dual tensor $\widetilde{F}^{\mu\nu}$ or with correcting the procedure of taking the massless limit.

In this Section, I first choose the field operator, Eq. (44), such that:

$$F^{i0}_{(+)}(\mathbf{p}) \quad = \quad E^i(\mathbf{p}) \quad , \quad F^{jk}_{(+)}(\mathbf{p}) = -\epsilon^{jkl} B^l(\mathbf{p}) \quad ; \tag{61}$$

$$F^{i0}_{(-)}(\mathbf{p}) \quad = \quad \tilde{F}^{i0}(\mathbf{p}) = B^i(\mathbf{p}) \quad , \quad F^{jk}_{(-)}(\mathbf{p}) = \tilde{F}^{jk}(\mathbf{p}) = \epsilon^{jkl} E^l(\mathbf{p}) \quad , \tag{62}$$

where $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$ is the tensor dual to $F^{\mu\nu}$; and $\epsilon^{\mu\nu\rho\sigma} = -\epsilon_{\mu\nu\rho\sigma}$, $\epsilon^{0123} = 1$ is the totally antisymmetric Levi-Civita tensor. After lengthy but standard calculations one achieves:*

$$\mathbf{J}^k = \sum_{\sigma\sigma'} \int \frac{d^3\mathbf{p}}{(2\pi)^3 2E_p} \left\{ \frac{i\epsilon^{ijk} \mathbf{E}^i_\sigma(\mathbf{p}) \mathbf{B}^j_{\sigma'}(\mathbf{p})}{2} \left[a(\mathbf{p},\sigma) b^\dagger(\mathbf{p},\sigma') + a(\mathbf{p},\sigma') b^\dagger(\mathbf{p},\sigma) + \right.\right.$$
$$+ \quad b^\dagger(\mathbf{p},\sigma') a(\mathbf{p},\sigma) + b^\dagger(\mathbf{p},\sigma) a(\mathbf{p},\sigma') \big] -$$

*Of course, the question of the behaviour of vectors $\mathbf{E}_\sigma(\mathbf{p})$ and $\mathbf{B}_\sigma(\mathbf{p})$ and/or of creation and annihilation operators with respect to the discrete symmetry operations in this particular case deserves detailed elaboration.

$$- \quad \frac{1}{2E_p}\left[i\mathbf{p}^k(\mathbf{E}_\sigma(\mathbf{p})\cdot\mathbf{E}_{\sigma'}(\mathbf{p})+\mathbf{B}_\sigma(\mathbf{p})\cdot\mathbf{B}_{\sigma'}(\mathbf{p}))-\right. \tag{63}$$
$$- \quad \left. i\mathbf{E}^k_{\sigma'}(\mathbf{p})(\mathbf{p}\cdot\mathbf{E}_\sigma(\mathbf{p})) - i\mathbf{B}^k_{\sigma'}(\mathbf{p})(\mathbf{p}\cdot\mathbf{B}_\sigma(\mathbf{p}))\right]\times\left[a(\mathbf{p},\sigma)b^\dagger(\mathbf{p},\sigma')+b^\dagger(\mathbf{p},\sigma)a(\mathbf{p},\sigma')\right]\Big\}$$

One should choose normalization conditions for field functions in the momentum representation. For instance, one can use the analogy with the (dual) classical electrodynamics:†

$$(\mathbf{E}_\sigma(\mathbf{p})\cdot\mathbf{E}_{\sigma'}(\mathbf{p})+\mathbf{B}_\sigma(\mathbf{p})\cdot\mathbf{B}_{\sigma'}(\mathbf{p})) = 2E_p\delta_{\sigma\sigma'} \quad , \tag{64}$$
$$\mathbf{E}_\sigma\times\mathbf{B}_{\sigma'} = \mathbf{p}\delta_{\sigma\sigma'} - \mathbf{p}\delta_{\sigma,-\sigma'} \quad . \tag{65}$$

These conditions still imply that $\mathbf{E}\perp\mathbf{B}\perp\mathbf{p}$.

Finally, one obtains

$$\mathbf{J}^k = -i\sum_\sigma\int\frac{d^3\mathbf{p}}{(2\pi)^3}\frac{\mathbf{p}^k}{2E_p}\left[a(\mathbf{p},\sigma)b^\dagger(\mathbf{p},-\sigma)+b^\dagger(\mathbf{p},\sigma)a(\mathbf{p},-\sigma)\right] . \tag{66}$$

If we want to describe states with the definite helicity quantum number (photons) we should assume that $b^\dagger(\mathbf{p},\sigma) = ia^\dagger(\mathbf{p},\sigma)$ which is reminiscent of the Majorana-like theories [35,38]. ‡ One can take into account the prescription of the normal ordering and set up the commutation relations in the form:

$$\left[a(\mathbf{p},\sigma), a^\dagger(\mathbf{k},\sigma')\right]_- = (2\pi)^3\delta(\mathbf{p}-\mathbf{k})\delta_{\sigma,-\sigma'} \quad . \tag{67}$$

After acting the operator (66) on the physical states, *e.g.*, $a^\dagger(\mathbf{p},\sigma)|0>$, we are convinced that the antisymmetric tensor field can describe particles with helicities to be equal to ± 1). One can see that an origin of this conclusion is the possibility of different definitions of the field operator (and its normalization), non-unique definition of the energy-momentum tensor [49,28,29] and possible existence of the '*antiparticle*' for the particle described by antisymmetric tensor field. This consideration is obviously related to the Weinberg discussion of the connection between helicity and representations of the Lorentz group [5a]. Next, I would like to point out that the Proca-like equations for antisymmetric tensor field with *mass*, *e.g.*, Eq. (47) can possess tachyonic solutions, see for the discussion in ref. [8]. Therefore, in a massive case the free physical states can be mixed with unphysical (at the present level of our knowledge) tachyonic states.

†Different choices of the normalization can still lead to equating the spin operator to zero or even to the other values of helicity, which differ from ± 1. This was also discussed with Prof. N. Mankoc-Borstnik during the Workshop "Lorentz Group, CPT and Neutrinos" (Zacatecas, México, June 23-26, 1999). The question is: which cases are realized in Nature and what processes do correspond to every case?

‡Of course, the imaginary unit can be absorbed by the corresponding re-definition of negative-frequency solutions.

4. Normalization and $m \to 0$ Limit of the Proca Theory

As opposed to the previous sections, where we assumed non-tentability of the application of the generalized Lorentz condition, in this Section we pay more attention to the correct procedure of taking the massless limit. We note that not all the obscurities were clarified in previous sections and recent works [55,56,42].[§] Let us analyze in a straightforward manner the operator (59). If one uses the following definitions of positive- and negative-energy parts of the antisymmetric tensor field in the momentum space, i. e., according to (27-32) with ($\alpha_\sigma = 0$):

$$(F_{\mu\nu})^{(+)}_{+1} = +(F_{\mu\nu})^{(-)}_{-1}\, , \, (F_{\mu\nu})^{(+)}_{-1} = +(F_{\mu\nu})^{(-)}_{+1}\, , \, (F_{\mu\nu})^{(+)}_{0} = -(F_{\mu\nu})^{(-)}_{0} \quad . \tag{68}$$

for the field operator (44) then one obtains in the frame where $\mathbf{p}^{1,2} = 0$:

$$\begin{aligned}
\mathbf{J}^k \;\; &\equiv \;\; \frac{m}{2}\int d^3\mathbf{x}\, \mathbf{E}(x^\mu)\times \mathbf{A}(x^\mu) = \frac{m^2}{4}\int \frac{d^3\mathbf{p}}{(2\pi)^3\, 4E_p^2}\left\{ \begin{pmatrix} 0\\ 0\\ E_p \end{pmatrix} \right. \\
& \qquad \big[a(\mathbf{p},+1)b^\dagger(\mathbf{p},+1) - a(\mathbf{p},-1)b^\dagger(\mathbf{p},-1) + \\
&+ \quad b^\dagger(\mathbf{p},+1)a(\mathbf{p},+1) - b^\dagger(\mathbf{p},-1)a(\mathbf{p},-1)\big] + \\
&+ \quad \frac{E_p}{m\sqrt{2}}\begin{pmatrix} E_p\\ iE_p\\ 0 \end{pmatrix}\big[a(\mathbf{p},+1)b^\dagger(\mathbf{p},0) + b^\dagger(\mathbf{p},-1)a(\mathbf{p},0)\big] + \\
&+ \quad \frac{E_p}{m\sqrt{2}}\begin{pmatrix} E_p\\ -iE_p\\ 0 \end{pmatrix}\big[a(\mathbf{p},-1)b^\dagger(\mathbf{p},0) + b^\dagger(\mathbf{p},+1)a(\mathbf{p},0)\big] + \\
&+ \quad \frac{1}{\sqrt{2}}\begin{pmatrix} m\\ -im\\ 0 \end{pmatrix}\big[a(\mathbf{p},0)b^\dagger(\mathbf{p},+1) + b^\dagger(\mathbf{p},0)a(\mathbf{p},-1)\big] + \\
&+ \quad \left. \frac{1}{\sqrt{2}}\begin{pmatrix} m\\ im\\ 0 \end{pmatrix}\big[a(\mathbf{p},0)b^\dagger(\mathbf{p},-1) + b^\dagger(\mathbf{p},0)a(\mathbf{p},+1)\big] \right\} .
\end{aligned} \tag{69}$$

Above, we used that according to dynamical equations (15,16) written in the momentum representation

$$[(\partial_\mu F^{\mu j}(\mathbf{p},\sigma)]^{(+)} = -\frac{m}{2}u^j(\mathbf{p},\sigma)\ ,\ [(\partial_\mu F^{\mu j}(\mathbf{p},\sigma)]^{(-)} = -\frac{m}{2}[u^j(\mathbf{p},\sigma)]^* \tag{70}$$

$$[\partial_\mu \widetilde{F}^{\mu j}(\mathbf{p},\sigma)]^{\pm} = 0\ . \tag{71}$$

[§]First of all, we note that the equality of the angular momentum generators to zero can be reinterpreted as

$$W_\mu P^\mu = 0\ ,$$

with W_μ being the Pauli-Lubanski operator. This yields

$$W_\mu = \lambda P_\mu$$

in the massless case. But, according to the analysis above the 4-vector W_μ would be equal to zero *identically* in the massless limit. This is not satisfactory from the conceptual viewpoints.

Next, it is obvious that though $\partial_\mu F^{\mu\nu}$ may be equal to zero in the massless limit from the formal viewpoint, and the equation (69) is proportional to the squared mass (?) at the first sight, it must not be forgotten that the commutation relations may provide additional mass factors in the denominator of (69). It is the factor $\sim E_p/m^2$ in the commutation relations¶

$$[a(\mathbf{p},\sigma), b^\dagger(\mathbf{k},\sigma')] \sim (2\pi)^3 \frac{E_p}{m^2} \delta_{\sigma\sigma'} \delta(\mathbf{p}-\mathbf{k}) \ . \tag{72}$$

which is required by the principles of the rotational and translational invariance‖ (and also by the necessity of the description of the Coulomb long-range force $\sim 1/r^2$ by means of the antisymmetric tensor field of the second rank).

The dimension of the creation/annihilation operators of the 4-vector potential should be $[\text{energy}]^{-2}$ provided that we use (24,25) with $N = m$ and $\epsilon^\mu \to u^\mu$. Next, if we want the $F^{\mu\nu}(x^\mu)$ to have the dimension $[\text{energy}]^2$ in the unit system $c = \hbar = 1$,* we must divide the Lagrangian by m^2 (with the same m, the mass of the particle!):

$$\mathcal{L} = \frac{\mathcal{L}(Eq.46)}{m^2} \,. \tag{73}$$

In this case, the antisymmetric tensor field has the dimension which is compatible with the inverse-square law, but the procedure of taking massless limit is somewhat different (and cannot be carried out from the beginning). This procedure will have the influence on the form of (59,69) because the derivatives in this case pick up the additional mass factor. Thus, we can remove the "ghost" proportionality of the c- number coefficients in (69) to $\sim m$. The commutation relations also change their form. For instance, one can now consider that $[a(\mathbf{p},\sigma), b^\dagger(\mathbf{k},\sigma')]_- \sim (2\pi)^3 2E_p \delta_{\sigma\sigma'}\delta(\mathbf{p}-\mathbf{k})$. The possibility of the above renormalizations was *not* noted in the previous papers on the theory of the 4-vector potential and of the antisymmetric tensor field of the second rank. Probably, this was the reason why people were confused after including the mass factor of the creation/annihilation operators in the field functions of $(1/2,1/2)$ and/or $(1,0)\oplus(0,1)$ representations, and/or applying the generalized Lorentz condition inside the dynamical invariant(s), which, as noted above, coincides in the form with the Maxwell free-space equations.

Finally, we showed that the interplay between definitions of field functions, Lagrangian and commutation relations occurs, thus giving the *non-zero* values of the angular momentum generators in the $(1,0)\oplus(0,1)$ representation.

¶Remember that the dimension of the δ function is inverse to its argument.

‖That is to say: the factor $\sim \frac{1}{m^2}$ is required if one wants to obtain non-zero energy (and, hence, helicity) excitations.

*The dimensions $[\text{energy}]^{+1}$ of the field operators for strengths was used in my previous papers in order to keep similarities with the Dirac case (the $(1/2,0)\oplus(0,1/2)$ representation) where the mass term presents explicitly in the term of the bilinear combination of the fields.

The conclusion of the "transversality" (in the meaning of existence of $\sigma = \pm 1$) is in accordance with the conclusion of the Ohanian's paper [55], cf. formula (7) there:*

$$\mathbf{J} = \frac{1}{2\mu_0 c^2} \int \Re e(\mathbf{E} \times \mathbf{A}^*) \, d^3\mathbf{x} = \pm \frac{1}{\mu_0 c^2} \int \frac{\hat{\mathbf{z}} E_0^2}{\omega} d^3\mathbf{x} \, , \tag{74}$$

with the Weinberg theorem, also with known experiments. The question, whether the situation could be realized when the spin of the antisymmetric tensor field would be equal to zero (in other words, whether the antisymmetric tensor field with unusual normalization exists or whether the third state of the massless 4-vector potential exists, as argued by Ogievetskiĭ and Polubarinov [20]), must be checked by additional experimental verifications. We do not exclude this possibility, founding our viewpoint on the papers [21,23,43,52].

Finally, one should note that we agree with the author of the cited work [55], see Eq. (4), about the gauge *non*-invariance of the division of the angular momentum of the electromagnetic field into the "orbital" and "spin" part (74). We proved again that for the antisymmetric tensor field $\mathbf{J} \sim \int d^3\mathbf{x}\, \mathbf{E} \times \mathbf{A}$. So, what people actually did (when spoken about the Ogievetskiĭ-Polubarinov-Kalb-Ramond field is: When $N = m$ they considered the gauge part of the 4-vector field functions. Then, they equated $\mathbf{A}$ containing the transverse modes on choosing $p_r = p_l = 0$ (see formulas (24)).† Under this choice the $\mathbf{E}(\mathbf{p}, 0)$ and $\mathbf{B}(\mathbf{p}, 0)$ are equal to zero in massless limit. But, the gauge part of $u^\mu(\mathbf{p}, 0)$ is not. The spin angular momentum can still be zero. When $N = 1$ the situation may be the same because of the different form of dynamical equations and the Lagrangian. So, for those who prefer simpler consideration it is enough to regard all possible states of 4-potentials/antisymmetric tensor field in the massless limit in the calculation of physical observables. Of course, I would like to repeat, it is not yet clear and it is not yet supported by reliable experiments whether the third state of the 4-vector potential/antisymmetric tensor field has physical significance and whether it is observable.

5. Conclusions

In conclusion, I calculated the Pauli-Lubanski vector of relativistic spin on the basis of the Nötherian symmetry method [48,49,50]. Let me recall that it is con-

*The formula (7) of ref. [55] is in the SI unit system and our arguments above are similar in the physical content. But, remember that in almost all papers the electric field is defined to be equal to $\mathbf{E}^i = F^{i0} = \partial^i A^0 - \partial^0 A^i$, with the potentials being not well-defined in the massless limit of the Proca theory. Usually, the divergent part of the potentials was referred to the gauge-dependent part. Furthermore, the physical fields and potentials were considered classically in the cited paper, so the integration over the 3-momenta (the quantization inside a cube) was not implied, see the formula (5) there. Please pay also attention to the complex conjugation operation on the potentials in the Ohanian's formula. We did not still exclude the possibility of the mathematical framework, which is different from our presentation, but the conclusions, in my opinion, must be in accordance with the Weinberg theorem.

†The reader, of course, can consider equating by the usual gauge transformation, $A^\mu \to A^\mu + \partial^\mu \chi$.

nected with the angular momentum vector, which is conserved as a consequence of the rotational invariance. After explicit [21] (or implicit [23]) applications of the constraints (the generalized Lorentz condition) in the Minkowski space, the antisymmetric tensor field becomes '*longitudinal*' in the meaning that the angular momentum operator is equated to zero (this interpretation was attached by the authors of the works [20,21,23]). I proposed one of the possible ways to resolve this apparent contradiction with the Correspondence Principle in refs. [8,9,10,11] and in several unpublished works[56]. The present article continues and sums up this research. The achieved conclusion is: the antisymmetric tensor field can describe both the Maxwellian $j = 1$ field and the Ogievetskiĭ-Polubarinov-Kalb-Ramond $j = 0$ field. Nevertheless, I still think that the physical nature of the $E = 0$ solution discovered in ref. [33], its connections with the so-called $\mathbf{B}^{(3)}$ field, ref. [7,52], with Avdeev-Chizhov δ'- type transversal solutions [23b], which cannot be interpreted as relativistic particles, as well as with my concept of χ boundary functions, ref. [11], are not completely explained until now. Finally, while I do not have any intention of doubting the theoretical results of the ordinary quantum electrodynamics, I am sure that the questions put forth in this note (as well as in previous papers of both mine and other groups) should be explained properly.

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